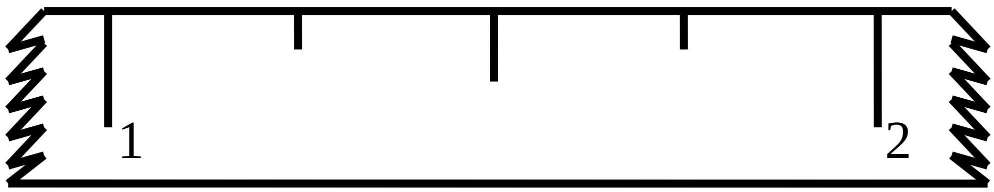
Lesson 1-4: Measuring Segments and Angles

Consider the following section of a ruler showing 1” and 2”:



How many points are there between the 1” and the 2” marks?

Did you say three? Don’t be fooled by the fact that only the ½” and ¼” marks are displayed. There are really an infinite number of points between 1” and 2”. Just keep dividing the gaps in half. You can keep going forever!

Now, what would you say the distance is between the following two points?
Assume it is in proportion with the ruler above…

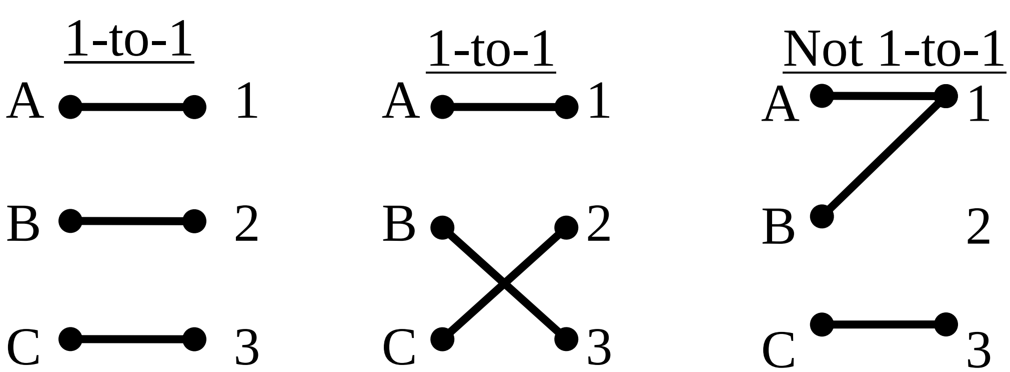
Did you estimate 1”? That would be correct. Did you subtract point *B*’s location on the ruler (2”) from point *A*’s location (1”)? B – A = 1”

Could you do A – B? Sure! Remember, distance is an absolute value of the difference between two points on a number line. So … │A - B│ = │ -1 │ = 1”.

Excellent, now a quick digression … hang with me here …

What does “one-to-one correspondence” mean?

It means you can pair every item in one set up with one and exactly one item in another set with none left over in either set…you use them all up.



Lesson 1-4: Measuring Segments and Angles

Postulate 1-5 (the Ruler Postulate)

Points on a line can be put into a *one-to-one correspondence* with the real number line so that the distance between any two points is the absolute value of the distance of the corresponding numbers.

Simply put … you can use a ruler (or any other straight measuring device) to find distance between points.

Seem like a rather obvious thing to state? Remember, we are building the system of geometry here…if we are going to assume something, we need to state it clearly and make sure it is true. Go back to lesson 1-2 and review what an axiom/postulate is.

Segment length

We represent the length of a segment $\overline{AB}$ as AB (no bar above the letters).

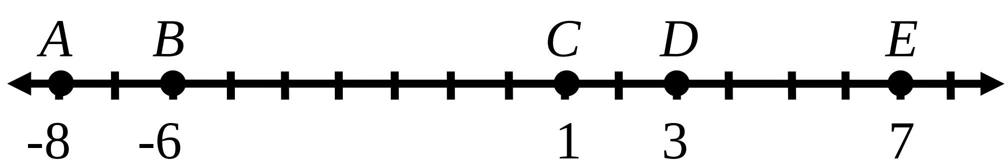
Given segment $\overline{AB}$ and the points a, b on the real number line corresponding to A and B, the length of $\overline{AB} = AB = |a - b|$

Definition

Congruent segments:

- Two segments with the same length.
- Represented by the symbol $\cong$
- Here is the relationship stated in "math":
    - If $AB = FE$ then $\overline{AB} \cong \overline{FE}$
- Here it is in English
    - If the lengths of segments AB and FE are equal, the segments are congruent.

Example: Pg 29, problem #2



Is $\overline{BD} \cong \overline{CE}$?

$BD = 9$
$CE = 6$

$9 \neq 6$

No, they are not congruent.

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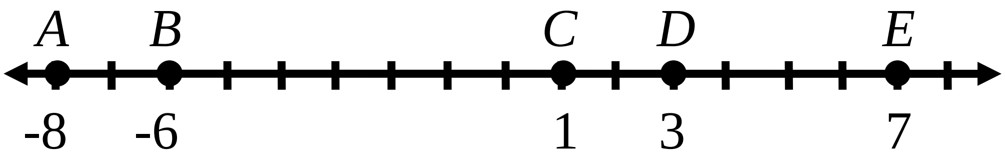
Now, look at $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ … how do they relate?



- In math:
  - $AB + BC = AC$
- In English:
  - The sum of the lengths of segments $\overline{AB}$ and $\overline{BC}$ equals the length of segment $\overline{AC}$.

Postulate 1-6 (the Segment Addition Postulate)

If point *B* is on $\overline{AC}$ and between points *A* and *C*, then $AB + BC = AC$.

This is a powerful postulate as you will see in the following example…

Example: Pg 29, problem #10

If $RS = 3x + 1$, $ST = 2x – 2$, $RT = 64$ …

- What is x?

  Using postulate 1-6 we know that $RS + ST = RT$, *so…*

  $$(3x + 1) + (2x – 2) = 64 \quad \text{(by substitution)}$$
  $$3x + 2x + 1 – 2 = 64$$
  $$5x – 1 = 64$$
  $$5x = 65$$
  $$x = 13$$

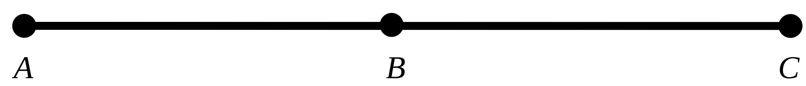
- What is *RS*?

  $RS = 3x + 1 = 3(13) + 1 = 40$

- What is *RT*?
  - $ST = 2x – 2 = 2(13) – 2 = 24$

Lesson 1-4: Measuring Segments and Angles

Consider the following:



If $AB = BC$, what would you call point B? It is the *midpoint* of the segment.

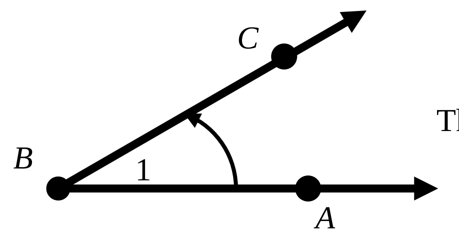
Definition

<u>Midpoint of a segment</u>:

- The point that divides the segment into two congruent segments.

Exploration

What would we have if two rays shared the same endpoint but were not collinear?



The two rays would form an *angle*.

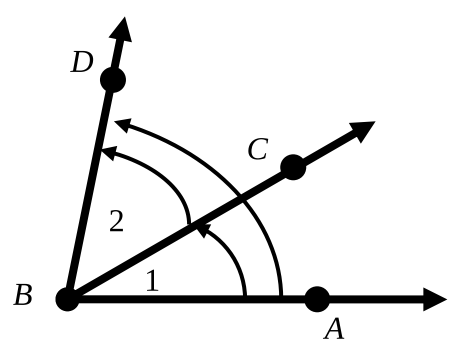
Definition

<u>Angle</u>:

- Two rays that share a common endpoint provided the two rays do not lie on the same line.
- The rays are the *sides* of the angle.
- The endpoint is the *vertex* of the angle.
- Represented by the symbol $\angle$
- Named by (see above diagram):
    - The three points defining the rays: $\angle ABC$ (vertex point always in the middle).
    - By the vertex point (if there is no ambiguity): $\angle B$
    - You can also number the angles and refer to them by the number: $\angle 1$

Exploration

In the following, does it make sense to name any of the angles $\angle B$?



No, there are 3 angles shown ($\angle ABD$, $\angle ABC$ & $\angle CBD$) … which one is it?

Lesson 1-4: Measuring Segments and Angles

Exploration

Remember the Ruler Postulate? We found a way of measuring segments by recognizing a 1-to-1 correspondence with a ruler.

Can you think of something we could do the same with so we can measure angles?

Postulate 1-7 (the Protractor Postulate)

Let rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be opposite rays in a plane.

Rays $\overrightarrow{OA}$, $\overrightarrow{OB}$ and all rays with endpoint O can be drawn on one side of $\overleftrightarrow{AB}$ can be pared with the real numbers from 0 to 180 so that:

1. $\overrightarrow{OA}$ is pared with 0 and $\overrightarrow{OB}$ is pared with 180
2. if $\overrightarrow{OC}$ is pared with $\boldsymbol{x}$ and $\overrightarrow{OD}$ is pared with $\boldsymbol{y}$, then $m\angle COD = |x - y|$

We note the measure of $\angle COD$ as $m\angle COD$.

Again, this simply means we can measure angles in degrees.

Now that we can measure angles, we can classify special types of angles.

Types of angles

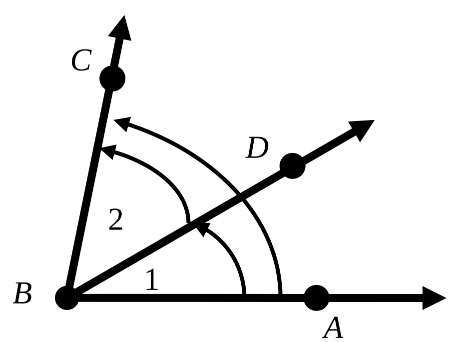
- Acute: $0° < \boldsymbol{x} < 90°$
- Right: $\boldsymbol{x} = 90°$
- Obtuse: $90° < \boldsymbol{x} < 180°$
- Straight: $\boldsymbol{x} = 180°$

Exploration

Remember the Segment Addition Postulate?
Given that we're talking about angles, what do you think our next postulate will be?

Postulate 1-8 (the Angle Addition Postulate)

If point D lies in the interior of $\angle ABC$,
then $m\angle ABD + m\angle DBC = \text{m}\angle ABC$



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Now, what do you imagine $\angle ABC \cong \angle DEF$ means?

Definition

<u>Congruent angles</u>:

- Angles with the same measure.
- If m$\angle COD = m\angle FGH$, then $\angle COD \cong \angle FGH$

Assign homework

p. 29 1-23 odd
25
27-35 odd
43-49 odd
60
66
70-72
75-78